V1994 AP. 1)

#1 $f(x) = 3x^4 + x^3 - 21x^2$

(a) $f'(x) = 12x^3 + 3x^2 - 42x$

$f'(2) = 96 + 12 - 84 = 24$

$\Rightarrow y + 28 = 24(x-2)$

$y + 28 = 24x - 48$

$\boxed{y - 24x + 76 = 0}$

(b) $f'(x) = 0$

$3x(4x^2 + x - 14) = 0$

$3x(4x-7)(x+2) = 0$

$x = 0 \quad x = 7/4 \quad x = -2$

$f''(x) = 36x^2 + 6x - 42$

$f''(0) < 0 \; ; \; f''(7/4) > 0$

$f''(-2) > 0$

$\Rightarrow$ Rel. Minima @ $x = 7/4,\ x = -2$

$f(7/4) = \frac{-9077}{256} \qquad f(-2) = -44$

Since $f(x) \to \infty$ for $x \to \pm\infty$

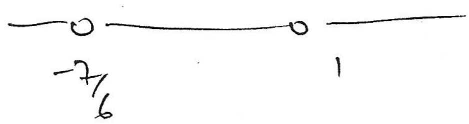
$\boxed{f(x)_{\text{Abs. Min}} = -44}$

(c) $f''(x) = 0$

$6(6x^2 + x - 7) = 0$

$6(6x+7)(x-1) = 0$

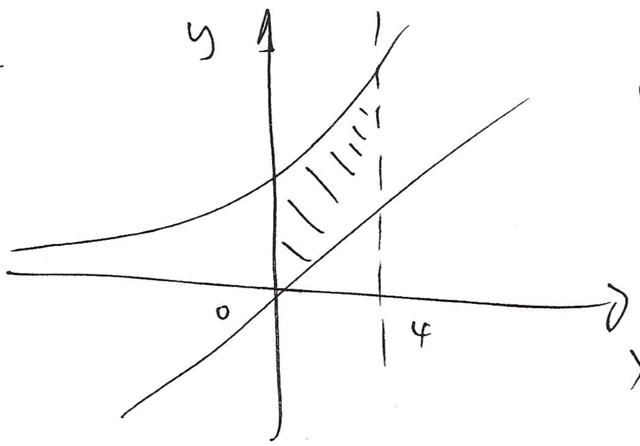
$x = -7/6 \quad x = 1$



$f''(x)$: $+ \quad - \quad +$

$\boxed{\text{Answer: } 1, -7/6}$

#2



(a) Area $= \int_0^4 e^x - x \, dx$

$= \left(e^x - \frac{x^2}{2}\right)_0^4$

$= e^4 - 8 - e^0$

$= e^4 - 9$ //

#2 (cont...)

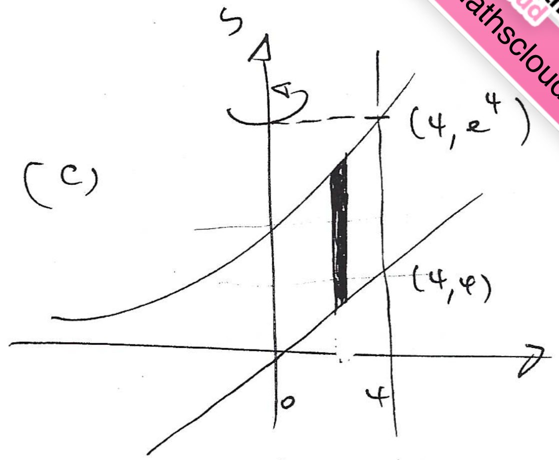
(b) $V = \pi \int_0^4 (e^x)^2 - (x)^2 \, dx$

$= \pi \left[\frac{1}{2}e^{2x} - \frac{x^3}{3} \right]_0^4$

$= \frac{\pi e^8}{2} - \frac{64\pi}{3} - \frac{\pi}{2}$

$= \frac{\pi}{6}\left[3e^8 - 131\right]$

(c)



$V = 2\pi \int_0^4 x.(e^x - x)\, dx$

SHELL

#3 $x^2 + xy + y^2 = 27$

(a) $2x + xy' + y + 2yy' = 0$

$(x+2y)y' = -2x - y$

$\boxed{y' = \frac{-2x-y}{x+2y}}$

(b) $y = 0$

$x^2 = 27$

$x = \pm 3\sqrt{3}$

slope @ $x = 3\sqrt{3}$, $y = 0$

$m_T = \frac{-6\sqrt{3}}{3\sqrt{3}} = -2$ m

slope @ $x = -3\sqrt{3}$, $y = 0$

$m_T = \frac{6\sqrt{3}}{-3\sqrt{3}} = -2$ m

$\Rightarrow$ tangents are parallel.

(c) y' is undefined when

$x = -2y$

$\Rightarrow 4y^2 - 2y^2 + y^2 = 27$

$y^2 = 9$

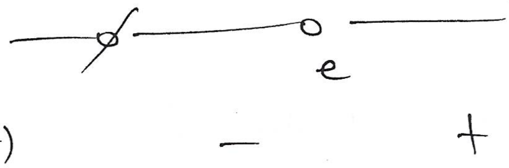
$y = \pm 3,$

4 $v(t) = t\ln t - t \quad , \quad t > 0$

(a) $a(t) = t \cdot \frac{1}{t} + \ln t - 1 = \boxed{\ln t}$

(b) condition: $v(t) > 0$

$t(\ln t - 1) > 0$

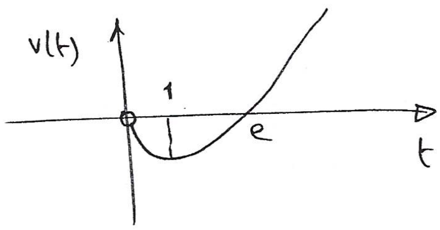


when $v(t) = 0$

$t \neq 0 \ , \ t = e$

$\boxed{\text{Ans: } t > e}$

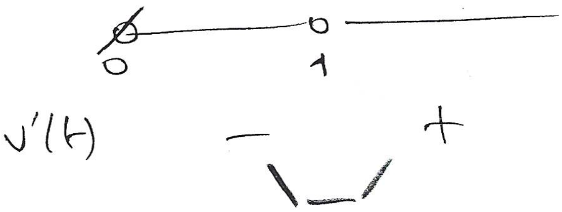
(c) V_{min} occurs when $v'(t) = a(t) = 0$



i.e $\ln t = 0$

$t = e^0 = 1$

$\Rightarrow \boxed{V_{min} = -1}$



(d) $x(t) = \int t\ln t - t \, dt$

$= \frac{t^2}{2}\ln t - \int \frac{t^2}{2} \cdot \frac{1}{t} \, dt - \frac{t^2}{2}$

$= \frac{t^2}{2}\ln t - \frac{t^2}{4} - \frac{t^2}{2} + c$ 3 6/4

$x(1) = 6 = 0 - \frac{1}{4} - \frac{1}{2} + c \Rightarrow c = 5\frac{1}{4} = \frac{21}{4}$

$\Rightarrow \boxed{x(t) = \frac{t^2}{2}\ln t - \frac{3t^2}{4} + \frac{21}{4}}$

4

$v(t) = t\ln t - t \quad , \quad t > 0$

(a) $a(t) = t \cdot \frac{1}{t} + \ln t - 1 = \boxed{\ln t}$

(b) condition: $v(t) > 0$

$t(\ln t - 1) > 0$

when $v(t) = 0$

$t \neq 0 \; , \; t = e$

$\boxed{\text{Ans: } \; t > e}$

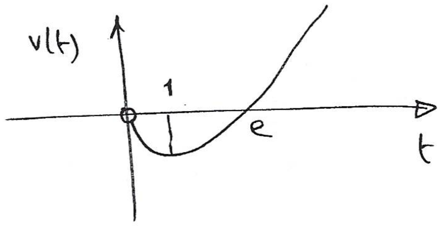
$\cancel{0}$ ——— e ———

$v(t)$ $\quad - \quad +$

(c) V_{min} occurs when $v'(t) = a(t) = 0$

i.e. $\ln t = 0$

$t = e^0 = 1$



$\Rightarrow \boxed{V_{min} = -1}$

$\cancel{0}$ ——— 1 ———

$v'(t)$ $\quad - \quad +$

(d) $x(t) = \int t\ln t - t \, dt$

$= \frac{t^2}{2}\ln t - \int \frac{t^2}{2} \cdot \frac{1}{t} \, dt - \frac{t^2}{2}$

$= \frac{t^2}{2}\ln t - \frac{t^2}{4} - \frac{t^2}{2} + C$ $\quad 6\frac{3}{4}$

$x(1) = 6 = 0 - \frac{1}{4} - \frac{1}{2} + C \Rightarrow C = 5\frac{1}{4} = \frac{21}{4}$

$\Rightarrow \boxed{x(t) = \frac{t^2}{2}\ln t - \frac{3t^2}{4} + \frac{21}{4}}$

#5 (a) $C = 2\pi R$

$$\frac{dC}{dt} = 2\pi \frac{dR}{dt}$$

$$\Rightarrow \frac{dR}{dt} = \frac{6}{2\pi} = \frac{3}{\pi}$$

$P = 8R$

$$\frac{dP}{dt} = 8\frac{dR}{dt} = \boxed{\frac{24}{\pi} \text{ inches/sec.}}$$

(b) $A = \pi R^2 = 25\pi$

$R > 0$, $R = 5$

A_E $A_{\text{ENCLOSED}} = 4R^2 - \pi R^2$

$$\frac{dA_E}{dt} = 8R.\frac{dR}{dt} - 2\pi R\frac{dR}{dt}$$

$$= 8.5.\frac{3}{\pi} - 2\cancel{\pi}.5.\frac{3}{\cancel{\pi}}$$

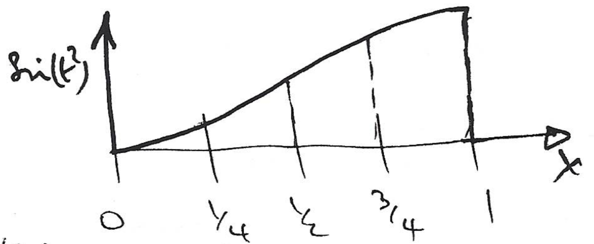
$$= \boxed{\frac{120}{\pi} - 30}$$

#6 (a) $F(1) = \int^1 \sin(t^2)\,dt$

$\int_0^1 \sin(t^2)\,dt \approx \frac{b-a}{2n}\left[\_\_\_ \; \_\_\_\right]$

$$\Rightarrow F(1) \approx \frac{1-0}{2(4)}\left\{0 + 2\left[\sin\left(\tfrac{1}{16}\right) + \sin\left(\tfrac{1}{4}\right) + \sin\left(\tfrac{9}{16}\right)\right] + \sin(1)\right\}$$

$\approx \boxed{0.314}$ (3dp)



$f(0) = 0$

$f(1/4) = \sin(1/16)$

$f(1/2) = \sin(1/4)$

$f(3/4) = \sin 9/16$

$f(1) = \sin 1$

(b) $F(x)$ is increasing when $F'(x) > 0$

i.e $\frac{d}{dx}\int_0^x \sin(t^2)\,dt > 0$

OR $\sin(x^2) > 0$ By the 2nd Fund. theorem of Calculus.

when $\sin x^2 = 0$

$x^2 = N\pi$, N is AN INTEGER

$\therefore x = \pm\sqrt{N\pi}$

$N=0$, $x = 0$. $N=2$, $x = \sqrt{2\pi}$ Since $0 \le x \le 3$

$N=1$, $x = \sqrt{\pi}$

$\Rightarrow \quad F'(x): \quad \underset{0}{\circ} \overset{+}{\text{——}} \underset{\sqrt{\pi}}{\circ} \overset{-}{\text{——}} \underset{\sqrt{2\pi}}{\circ} \overset{+}{\text{——}} \underset{3}{\circ}$

$\Rightarrow$ $F'(x)$ is INCREASING on $[0, \sqrt{\pi}) \cup (\sqrt{2\pi}, 3]$

(c) The MEAN VALUE THEOREM GIVES

$$F'(c) = \frac{F(3) - F(1)}{3-1} = K$$

$$F(3) = \int_0^3 \sin(t^2)\,dt \qquad F(1) = \int_0^1 \sin(t^2)\,dt$$

$$\Rightarrow \quad F(3) - F(1) = \int_0^3 \sin(t)^2\,dt - \int_0^1 \sin(t^2)\,dt$$

$$= \int_1^3 \sin(t^2)\,dt$$

$$\therefore \quad \int_1^3 \sin(t^2)\,dt = \boxed{2K} \text{ —— Answer.}$$

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